LESSON 1.1 Rational and Irrational Numbers

COMMON CORE 8.NS.1

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; ... *Also 8.NS.2, 8.EE.2*

ESSENTIAL QUESTION

How do you rewrite rational numbers and decimals, take square roots and cube roots, and approximate irrational numbers?

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Expressing Rational Numbers as Decimals

A **rational number** is any number that can be written as a ratio in the form $\frac{a}{b}$, where a and b are integers and b is not 0. Examples of rational numbers are 6 and 0.5.

6 can be written as $\frac{6}{1}$.   0.5 can be written as $\frac{1}{2}$.

Every rational number can be written as a terminating decimal or a repeating decimal. A **terminating decimal**, such as 0.5, has a finite number of digits. A **repeating decimal** has a block of one or more digits that repeat indefinitely.

EXAMPLE 1

COMMON CORE 8.NS.1

Write each fraction as a decimal.

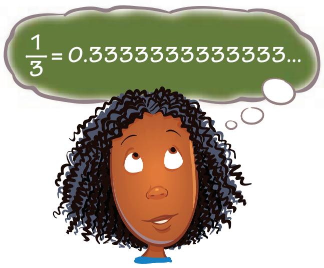
A $\frac{1}{4}$

$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \\ \underline{-8} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

Remember that the fraction bar means "divided by." Divide the numerator by the denominator.

Divide until the remainder is zero, adding zeros after the decimal point in the dividend as needed.

$\frac{1}{4} = 0.25$



B $\frac{1}{3}$

$$\begin{array}{r} 0.333 \\ 3\overline{)1.000} \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 1 \end{array}$$

Divide until the remainder is zero or until the digits in the quotient begin to repeat.

Add zeros after the decimal point in the dividend as needed.

When a decimal has one or more digits that repeat indefinitely, write the decimal with a bar over the repeating digit(s).

$\frac{1}{3} = 0.\overline{3}$

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My Notes

YOUR TURN

Write each fraction as a decimal.

1. $\frac{5}{11}$ ________ **2.** $\frac{1}{8}$ ________ **3.** $2\frac{1}{3}$ ________

Expressing Decimals as Rational Numbers

You can express terminating and repeating decimals as rational numbers.

EXAMPLE 2

COMMON CORE 8.NS.1

Write each decimal as a fraction in simplest form.

A 0.825

The decimal 0.825 means "825 thousandths." Write this as a fraction.

$\frac{825}{1000}$ *To write "825 thousandths", put 825 over 1000.*

Then simplify the fraction.

$\frac{825 \div 25}{1000 \div 25} = \frac{33}{40}$ *Divide both the numerator and the denominator by 25.*

$0.825 = \frac{33}{40}$

B $0.\overline{37}$

Let $x = 0.\overline{37}$. The number $0.\overline{37}$ has 2 repeating digits, so multiply each side of the equation $x = 0.\overline{37}$ by 10^2, or 100.

$x = 0.\overline{37}$

$(100)x = 100(0.\overline{37})$

$100x = 37.\overline{37}$ *100 times $0.\overline{37}$ is $37.\overline{37}$.*

Because $x = 0.\overline{37}$, you can subtract x from one side and $0.\overline{37}$ from the other.

$100x = 37.\overline{37}$

$-x \quad -0.\overline{37}$

$99x = 37$ *$37.\overline{37}$ minus $0.\overline{37}$ is 37.*

Now solve the equation for x. Simplify if necessary.

$\frac{99x}{99} = \frac{37}{99}$ *Divide both sides of the equation by 99.*

$x = \frac{37}{99}$

YOUR TURN

Write each decimal as a fraction in simplest form.

4. 0.12 ____________ **5.** $0.\overline{57}$ ____________ **6.** 1.4 ____________

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Finding Square Roots and Cube Roots

The **square root** of a positive number p is x if $x^2 = p$. There are two square roots for every positive number. For example, the square roots of 36 are 6 and -6 because $6^2 = 36$ and $(-6)^2 = 36$. The square roots of $\frac{1}{25}$ are $\frac{1}{5}$ and $-\frac{1}{5}$. You can write the square roots of $\frac{1}{25}$ as $\pm\frac{1}{5}$. The symbol $\sqrt{}$ indicates the positive, or **principal square root**.

A number that is a **perfect square** has square roots that are integers. The number 81 is a perfect square because its square roots are 9 and -9.

The **cube root** of a positive number p is x if $x^3 = p$. There is one cube root for every positive number. For example, the cube root of 8 is 2 because $2^3 = 8$. The cube root of $\frac{1}{27}$ is $\frac{1}{3}$ because $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$. The symbol $\sqrt[3]{}$ indicates the cube root.

A number that is a **perfect cube** has a cube root that is an integer. The number 125 is a perfect cube because its cube root is 5.

EXAMPLE 3

COMMON CORE 8.EE.2

Solve each equation for *x*.

A $x^2 = 121$

$x^2 = 121$ — *Solve for x by taking the square root of both sides.*

$x = \pm\sqrt{121}$ — *Apply the definition of square root.*

$x = \pm 11$ — *Think: What numbers squared equal 121?*

The solutions are 11 and -11.

B $x^2 = \frac{16}{169}$

$x^2 = \frac{16}{169}$ — *Solve for x by taking the square root of both sides.*

$x = \pm\sqrt{\frac{16}{169}}$ — *Apply the definition of square root.*

$x = \pm\frac{4}{13}$ — *Think: What numbers squared equal $\frac{16}{169}$?*

The solutions are $\frac{4}{13}$ and $-\frac{4}{13}$.

Math Talk
Mathematical Practices

Can you square an integer and get a negative number? What does this indicate about whether negative numbers have square roots?

C $729 = x^3$

$\sqrt[3]{729} = \sqrt[3]{x^3}$ — *Solve for x by taking the cube root of both sides.*

$\sqrt[3]{729} = x$ — *Apply the definition of cube root.*

$9 = x$ — *Think: What number cubed equals 729?*

The solution is 9.

D $x^3 = \frac{8}{125}$

$\sqrt[3]{x^3} = \sqrt[3]{\frac{8}{125}}$ — *Solve for x by taking the cube root of both sides.*

$x = \sqrt[3]{\frac{8}{125}}$ — *Apply the definition of cube root.*

$x = \frac{2}{5}$ — *Think: What number cubed equals $\frac{8}{125}$?*

The solution is $\frac{2}{5}$.

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YOUR TURN

Solve each equation for *x*.

7. $x^2 = 196$ __________

8. $x^2 = \frac{9}{256}$ __________

9. $x^3 = 512$ __________

10. $x^3 = \frac{64}{343}$ __________

EXPLORE ACTIVITY

COMMON CORE **8.NS.2, 8.EE.2**

Estimating Irrational Numbers

Irrational numbers are numbers that are not rational. In other words, they cannot be written in the form $\frac{a}{b}$, where *a* and *b* are integers and *b* is not 0. Square roots of perfect squares are rational numbers. Square roots of numbers that are not perfect squares are irrational. The number $\sqrt{3}$ is irrational because 3 is not a perfect square of any rational number.

Estimate the value of $\sqrt{2}$.

A Since 2 is not a perfect square, $\sqrt{2}$ is irrational.

B To estimate $\sqrt{2}$, first find two consecutive perfect squares that 2 is between. Complete the inequality by writing these perfect squares in the boxes.

$\square < 2 < \square$

C Now take the square root of each number.

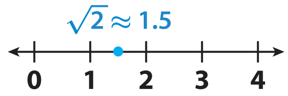
$\sqrt{\square} < \sqrt{2} < \sqrt{\square}$

D Simplify the square roots of perfect squares.

$\square < \sqrt{2} < \square$

$\sqrt{2}$ is between ______ and ______.

E Estimate that $\sqrt{2} \approx 1.5$.



F To find a better estimate, first choose some numbers between 1 and 2 and square them. For example, choose 1.3, 1.4, and 1.5.

$1.3^2 =$ ______ $1.4^2 =$ ______ $1.5^2 =$ ______

Is $\sqrt{2}$ between 1.3 and 1.4? How do you know?

____

____

Is $\sqrt{2}$ between 1.4 and 1.5? How do you know?

____

____

$\sqrt{2}$ is between ______ and ______, so $\sqrt{2} \approx$ ______.

G Locate and label this value on the number line.

Reflect

11. How could you find an even better estimate of $\sqrt{2}$?

____

____

12. Find a better estimate of $\sqrt{2}$. Draw a number line and locate and label your estimate.

$\sqrt{2}$ is between ______ and ______, so $\sqrt{2} \approx$ ______.

13. Estimate the value of $\sqrt{7}$ to two decimal places. Draw a number line and locate and label your estimate.

$\sqrt{7}$ is between ______ and ______, so $\sqrt{7} \approx$ ______.