



Warm-Up 1

- _____ If Franco types *MATHCOUNTS* 50 times in total, how many more consonants does he type than vowels?
- _____ seconds Vicki and Candace start a race at the same time. Vicki finishes in 28 minutes 47 seconds. Candace finishes in 29 minutes 46 seconds. How many seconds ahead of Candace does Vicki finish?
- _____ degrees What is the sum of the degree measures of the interior angles of a regular octagon?
- _____ \$ Vera's favorite coffee blend costs \$1.32 per ounce, including tax. How much will it cost Jorge to buy Vera one and a half pounds of her favorite coffee?
- _____ ways Lily is going to the movies with Abby, Bea and Jaclyn. Abby wants to sit at the end of a row, and Bea only cares that she is seated next to Jaclyn. In how many different ways can the girls be seated in a single row that has only four seats?
- _____ On the first five math quizzes of the school year, Maurice's scores were 80, 84, 99, 92 and 100. After the sixth quiz, Maurice's median score for all six quizzes was 90. What was Maurice's score on the sixth quiz?
- _____ regions What is the maximum number of non-overlapping regions that can be determined by three lines in a plane?
- _____ If $3x + 155 = 272$, then what is the value of $3x + 160$?
- _____ Ash's secret number is a factor of 72. The secret number is neither prime nor a multiple of 3. What is the sum of all possible values of Ash's secret number?
- _____ One-half of three-fourths of a number is 20 more than two-fifths of the same number. What is the number?



Warm-Up 2

11. _____ What is the least possible sum of the digits displaying the time on a 12-hour digital clock?

12. _____ palin-
dromes How many 4-digit palindromes contain both the digits 1 and 2?

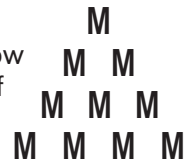
13. _____ cm What is the radius of a circle with a circumference measuring 24π cm?

14. _____ On a number line, what number is two-thirds of the distance from one-half to 2.25? Express your answer as a common fraction.


15. _____ m² The length of a rectangular sports field is three times its width. If the perimeter of the field is 880 meters, what is the area of the field?

16. _____ An item with an original price of d dollars has its price increased by x percent in April and then decreased by x percent in May. The resulting price is 4 percent less than the original price. What is the value of x ?

17. _____ rows Letter M s are stacked, as shown, so that the top row has one M , the second row has two M s, the third row has three M s, and so on. What is the least number of rows required for the total number of stacked M s to be divisible by 7?



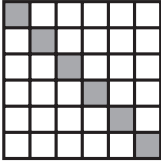
18. _____ minutes Josh mowed one-third of a 2000-ft² lawn in 18 minutes. At the same rate, how many minutes would it take him to mow a 4200-ft² lawn? Express your answer to the nearest whole number.

19. _____ units²  Arc XYZ, shown here, is a semicircle. If $XZ = 15$ units, what is the value of $(XY)^2 + (YZ)^2$?

20. _____ Seven contestants enter a drawing that begins with 100 balls numbered 1 through 100 in a box. Each contestant randomly selects a ball without replacement. The two contestants who select balls with the two highest numbers each will win a cash prize. The first six contestants select balls numbered 83, 5, 44, 67, 21 and 30. What is the probability that the last contestant will win a cash prize? Express your answer as a common fraction.

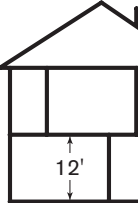


Warm-Up 3

21. _____ shifts A shift consists of rotating a two-digit number 90 degrees clockwise and then exchanging the position of the two digits. How many shifts must be performed before the number 19 returns to its original orientation?
22. _____ people The Population Reference Bureau reported a net gain of 155 people on Earth each minute. At this rate, how many more people are there on Earth every day?
23. _____ If $\frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} = 24$, what is the value of x ? Express your answer as a common fraction.
24. _____ times Maximo wrote down two rows of numbers as shown. In the first row, he wrote the positive multiples of 7, starting with 7 and ending with 700. In the second row, he wrote the positive multiples of 13, starting with 13 and ending with 1300. How many times did corresponding pairs of terms in the first and second rows have the same units digit?
- | | | | | | | | |
|-------------|----|----|----|----|----|-----|------|
| First row: | 7 | 14 | 21 | 28 | 35 | ... | 700 |
| Second row: | 13 | 26 | 39 | 52 | 65 | ... | 1300 |
25. _____ In the 6×6 array of squares shown, two squares are adjacent if they share a side. What is the probability that two adjacent squares, chosen at random, are not the same color? Express your answer as a common fraction.
- 
26. _____ integers A *very round number* is a positive integer that has exactly one nonzero digit. How many integers less than one billion are very round numbers?
27. _____ Adding $\frac{3}{8}$ to a number increases it by 5% of its original value. What is the original number? Express your answer as a decimal to the nearest tenth.
28. _____ cm What is the height of a right rectangular prism with a length of 4 cm, a width of 3 cm and a volume of 108 cm^3 ?
29. _____ minutes Ms. Swift wrote 40 pages of notes, each page containing approximately 240 words. Chanel, who types 36 words per minute, volunteered to type some of Ms. Swift's notes, beginning at the first page and progressing through them in order. Marcus, who types 54 words per minute, volunteered to type some of Ms. Swift's notes, beginning at the last page and progressing through them in reverse order. If Chanel and Marcus begin at the same time, working together, how many minutes will it take them to finish typing the notes? Express your answer to the nearest whole number.
30. _____ days An exponentially talented salesman sells 1 car on his first day, 2 cars on his second day, 4 cars on his third day and so on, so that every day after the first, he sells twice as many cars as the day before. How many days does it take him to sell a total of at least 1000 cars?



Workout 1

31. _____ m On average, a human fingernail grows at a rate of 3 mm per month. At this rate, how many total meters will Deshawn's 10 fingernails grow over the next 10 years? Express your answer as a decimal to the nearest tenth.
32. _____ % What percent of the prime numbers less than 100 have a units digit of 3?
33. _____ inches In Mrs. Abel's class, there are 16 girls and 14 boys. The mean of the girls' heights is 63.7 inches, and the mean of the boys' heights is 65.4 inches. What is the mean of the heights of all 30 students? Express your answer as a decimal to the nearest tenth.
34. \$ _____ The price P , in dollars, that AppCo charges annual subscribers for downloading x smartphone applications is calculated using the formula $P = 5 + 1.25x$. If a subscriber downloaded 25 applications during a year, what was the average cost per application?
35. _____ members The local gaming club has more than 2 but fewer than 20 members. Every member of the group purchased a signed copy of the latest book from the legendary Dumas, for a grand total of exactly \$299. If each member paid the same whole number dollar amount, how many members are in the club?
36. _____ inches Jamee is building a two-story house in which the top floor is 12 feet above the bottom floor. The local building code specifies that the height of a step cannot exceed 7.5 inches. What is the maximum height of a step in a staircase that Jamee can construct between the two floors if all steps will have the same height? Express your answer as a decimal to the nearest tenth.
- 
37. _____ mm The first and second books of a book series have 10 and 11 chapters, respectively, and each chapter has exactly 20 pages. The total thickness of the pages in the first book is 14 mm. For book two, the total thickness of the pages plus the front and back covers is 17 mm. The thickness of each page in book one is the same as that in book two, and the thickness of each cover in book one is the same as that of book two. What is the total thickness when the two books are stacked cover to cover? Express your answer as a decimal to the nearest tenth.
38. _____ The line that contains $(4, -7)$ and $(-3, 14)$ also contains the point $(0, b)$. What is the value of b ?
39. _____ What is the greatest multiple of 10 that can be expressed using only the numbers 2, 2, 3 and 5, each exactly once, with any or all of the operations $+$, $-$, \times and \div , exponentiation and parentheses? Express your answer in scientific notation.
40. _____ cm Two sides of a triangle are 15 cm and 18 cm in length. The altitude to the 18-cm side is 10 cm. What is the length of the altitude to the 15-cm side?



Workout 2

41. _____ triangles Point A is a vertex of a regular octagon. When all possible diagonals are drawn from point A in the polygon, how many triangles are formed?

42. _____ in² Alicia is framing a rectangular photo that measures 8 inches high by 12 inches wide. However, the frame she has was designed to hold a picture with sides in the ratio 4:5. If the frame Alicia has is the smallest possible frame that holds the photo, how much extra area will there be? Express your answer as a decimal to the nearest tenth.

43. \$ _____ If four hamburgers and two hot dogs cost \$16.40, and six hamburgers and four hot dogs cost \$26.40, what is the combined cost of a hamburger and a hot dog?

44. _____ % At Grace Hopper Middle School, there are 531 students in the sixth, seventh and eighth grades combined. The table below shows the number of students in each grade who play soccer and the number of students in each grade who do not play soccer. If all of the seventh and eighth graders attend an assembly, and two students at the assembly are chosen at random, what is the probability, as a percent, that neither student plays soccer? Express your answer to the nearest tenth.

Grade	Play Soccer	Don't Play Soccer
6	79	118
7	61	116
8	54	103

45. \$ _____ It costs \$0.12 for the Cheapco Soda Company to manufacture enough cola to fill a cylindrical container that is 2 inches in diameter and 4 inches in height. How much would it cost for Cheapco to manufacture enough cola to fill a cylindrical container that is 6 inches in diameter and 10 inches in height?

46. _____ minutes Kim created decorations for the school dance. It took her 4 minutes to create the first decoration, and each decoration after the first one took 10% less time to create than the one before it. How many minutes did it take her to create the first five decorations? Express your answer as a decimal to the nearest tenth.

47. _____ cm Lily made a circle from a length of string measuring 26 cm, and Willow made a circle from a length of string measuring 31 cm. What is the absolute difference in the radii of their circles? Express your answer as a decimal to the nearest tenth.

48. _____ The digits of the addends in the sum shown are represented by the letters A, B and G. What is the value of $A \times (B + G)$?

$$\begin{array}{r} G \ A \ B \\ + \ B \ A \ G \\ \hline 1 \ 0 \ 9 \ 0 \end{array}$$

49. _____ ways In how many ways can the integers 1 through 6 be written horizontally in a row so that the sum of any two adjacent integers is odd?

50. _____ units What is the radius of the largest sphere that will fit inside a cube of volume 8 units³?

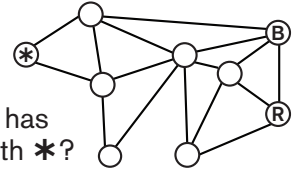


Warm-Up 4

51. _____ What is the smallest integer that is greater than the product $2.4999 \times 3.9999 \times 4.9999$?

52. _____ m² What is the area of a rectangle with sides that measure $2\frac{1}{2}$ and $7\frac{1}{3}$, in meters? Express your answer as a mixed number.

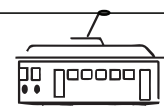
53. _____ In the figure, each circle is a vertex of one or more triangles. The circles marked B and R are colored blue and red, respectively. If each of the remaining circles is to be colored red, blue or yellow so that no triangle has two vertices of the same color, what is the color of the circle marked with *?



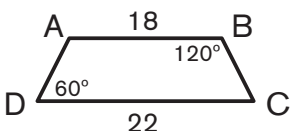
54. _____ The mean of the first five terms of an arithmetic sequence is 27, and the mean of the first eight terms of the same sequence is 39. What is the absolute difference between the first and second terms of this sequence?

55. _____ people What is the minimum number of people that must be in a room to guarantee that at least one person has a birthday whose day is a single digit, assuming that no two people have the same birthday?

56. _____ feet A cable car is 30 feet in length and travels back and forth in a straight line, on a single cable that is 3000 feet in length. Let P be a point somewhere on the cable car. During one round trip, what is the distance traveled by point P ?



57. _____ To cater a party, Fancy Caterers charges a certain amount per guest plus a fixed delivery fee. The total cost for a party with 25 guests is twice the cost of a party with 10 guests. If the cost of a party with n guests is twice the cost of a party with 25 guests, what is the value of n ?

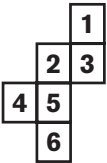
58. _____ cm  ABCD is a trapezoid with bases $AB = 18$ cm and $DC = 22$ cm. The measures of $\angle ABC$ and $\angle ADC$ are 120 degrees and 60 degrees, respectively. What is the perimeter of trapezoid ABCD?

59. _____ seconds Kaci's jogging speed is 60% faster than her walking speed. The time it takes her to walk a mile is $5\frac{1}{2}$ minutes longer than the time it takes her to jog a mile. How many seconds does it take Kaci to jog a mile?

60. _____ % Thirty percent of 80% of 20 is the same as what percent of 60% of 40?



Warm-Up 5

61. _____ coins Zoey has \$2.90 in quarters, dimes and nickels. The number of nickels is one more than six times the number of quarters, and the number of dimes is four times the number of quarters. What is the total number of coins Zoey has?
62. _____ days Every day, a cafeteria offers pizza, chicken and salad as lunch entrées. This comes with a choice of milk, juice, water or tea to drink and a cookie or brownie for dessert. If a different combination of one entrée, one drink and one dessert is tried every day, how many days does it take to try every combination?
63. _____ prisms A pile of clay can be molded to form a solid cube with edges of length 10 cm. How many solid rectangular prisms with dimensions $2 \times 4 \times 5$ cm can be made from the same amount of clay?
64. _____ \$ A restaurant offers an incentive by giving a \$5 coupon for every \$40 spent during a calendar year. If Roberto spends \$639 at the restaurant in a year, what is the total dollar value of the coupons he can expect to receive?
65. _____ golf balls GG's shipping company ships golf balls in three different box sizes. The small box contains a dozen golf balls. The medium box is similar to the small box, but each dimension is doubled. The large box also is similar to the small box, but each dimension is tripled. If Jake orders one box of each size, how many golf balls will he receive?
66. _____  This figure, composed of six congruent squares labeled 1 through 6, is a net of a cube. If it is folded to form a cube, what is the sum of the digits on the four faces that are adjacent to the face labeled 1?
67. _____ Two distinct numbers are selected at random from the set $\{1, 2, 3, 4, 5, 6\}$. What is the probability that their product is an odd number? Express your answer as a common fraction.
68. _____ If today is Friday, what day of the week will it be 101 days from today?
69. _____ base Thomas learned to count in a base other than 10. Instead of writing 163, which is in base 10, Thomas writes 431. What base is Thomas using?
70. _____ units Starting at the origin of a coordinate plane, an ant crawls 1 unit to the right, 2 units up, 3 units to the right, 4 units up, 5 units to the right and 6 units up. How far from the origin is the ant currently located?



Warm-Up 6

71. _____ When an integer is doubled and increased by 3, the result is 5 less than the square of the integer. What is the sum of all such integers?

72. _____ degrees Triangle ABC is isosceles with $AB = AC$ and $m\angle A = 30$ degrees. Side AB is extended to D so that $m\angle ACD = 90$ degrees. What is the degree measure of $\angle BCD$?

73. _____ games If the 15 teams in a soccer league each play eight games in a season, what is the total number of games played during the season?

74. _____ miles Laree notices that the current mileage on her car is a multiple of 1000 and is a perfect cube. She does some calculations and determines that she will have to drive another 4921 miles before the number of miles on her car is a perfect cube again. What is Laree's current mileage?

75. _____ units² Two lines parallel to the sides of a large rectangle divide the rectangle into four regions. The areas of three of the regions, starting in the upper right corner and going counterclockwise, are 24, 40 and 15 units² as shown (not to scale). What is the area of the large rectangle?

40	24
15	

76. _____ The first three terms of an arithmetic sequence are $17x + 20$, $18x - 3$ and $20x + 1$, in that order. What is the value of x ?

77. _____ mg The effectiveness of Donovan's cold medication decreases geometrically, retaining one-fourth of its original effectiveness after four hours. If Donovan takes 500 mg of medication every four hours beginning at 8:00 a.m., how much effective medication remains in his body at 6:00 p.m.? Express your answer as a mixed number.

78. _____ marbles Ron and Martin are playing a game with a bowl containing 39 marbles. Each player takes turns removing 1, 2, 3 or 4 marbles from the bowl. The person who removes the last marble loses. If Ron takes the first turn to start the game, how many marbles should he remove to guarantee he is the winner?

79. _____ What common fraction is equal to the sum $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{512}$?

80. \$ Mrs. Lowe is buying lunch for her class of 25 students. A large pizza that serves three people costs \$8, and a giant sub that serves four people costs \$9. If pizzas and subs cannot be purchased in part, what is the least amount it will cost Mrs. Lowe to feed all the students in her class?



Workout 3

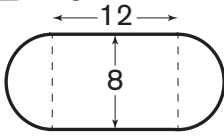
81. _____ terms How many terms of the sequence $3^1, 3^2, 3^3, \dots, 3^{100}$ have a units digit of 7?
82. _____ miles A car service charges \$2.40 for the first quarter of a mile traveled and then charges \$0.40 for each additional fifth of a mile. How many miles can a rider travel for \$10? Express your answer as a decimal to the nearest hundredth.
83. _____ Billie is downloading a video game to her computer. When she checks at 11:15 a.m., her computer indicates that the download is 35% complete. When she checks again at 11:30 a.m., the download is 80% complete. Assuming the video game is downloaded at a constant rate, Billie should expect the download to be complete after an additional a minutes b seconds. If a and b are whole numbers and $b < 60$, what is the value of $a \times b$?
84. _____ % On average, one out of every 70 widgets manufactured has a defect. If Nick inspects four widgets chosen at random, what is the probability, expressed as a percent, that at least one of them will be defective? Express your answer to the nearest tenth.
85. _____ inches A newly minted half-dollar coin has a diameter of 1.205 inches and 150 equally spaced ridges around its circumference. What is the distance between any two adjacent ridges? Express your answer as a decimal to the nearest thousandth.
86. _____ years
old Margo is currently twice as old as Joy was three decades ago, and Joy is currently three times as old as Margo was two decades ago. How old was Joy when Margo was born?
87. _____ in² What is the area of a rhombus with side length 10 inches and diagonal lengths that differ by 4 inches?
88. _____ % Octavio bought a very unpredictable stock. Its value increased by 10% each day on Monday, Tuesday and Wednesday, was unchanged on Thursday, then dropped by 30% on Friday. Octavio mistakenly thought that the 30% loss was offset completely by the three 10% gains. By what percent did the stock in fact decrease for the week? Express your answer to the nearest hundredth.
89. _____ inches Tootie Frootie candy is cylindrical in shape and comes in two different sizes. One size is 1.25 inches long with a circumference of 2 inches, and the other is 3.25 inches long and has twice the volume of the first. What is the circumference of the larger size? Express your answer as a decimal to the nearest hundredth.
90. _____ truck-
loads A 32-foot by 15-foot by 8-foot hole is being dug in Winnie's yard for a rectangular swimming pool. If the truck hauling away the dirt holds a maximum of 6 cubic yards, how many truckloads of dirt will be taken away? Express your answer as a whole number.



Workout 4

91. _____ quarters Laurie had 30 coins, all dimes and quarters, in a jar. Every morning for five days, she took out a quarter; then she replaced it with a dime when she got home from school. After the five days, Laurie had twice as many dimes as quarters. How many quarters were initially in Laurie's jar?

92. _____ yd³ A garden has the shape of a rectangle with semicircles on either end as shown. The length of the rectangular portion is 12 feet, and the width is 8 feet. How many cubic yards of topsoil are needed to fill the entire garden uniformly to a depth of 2 inches? Express your answer as a decimal to the nearest tenth.



93. _____ Two positive numbers a and b have geometric mean x if $x > 0$ and $\frac{a}{x} = \frac{x}{b}$. If 9 is the geometric mean of m and $m + 3.3$, what is the value of m ? Express your answer as a decimal to the nearest tenth.

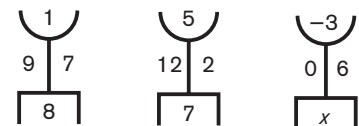
94. _____ mi/h On Oscar's trip to school, he took 50 minutes to walk 2.7 miles. He walked at a constant speed for the first 40 minutes, then increased his speed by exactly 1 mi/h. How fast was Oscar walking for the first 40 minutes of his trip? Express your answer as a decimal to the nearest hundredth.

95. _____ minutes Asanji and Allen are hired to paint the fence around a neighbor's property. Working alone, Allen can paint the entire fence in 9 hours, and Asanji can paint the same fence, working alone, in 7 hours. Working together, how many minutes will it take them to paint two-thirds of the fence? Express your answer as a decimal to the nearest tenth.

96. _____ miles

Airplane A flies 45 degrees east of north at a constant speed of 300 mi/h. Traveling at the same altitude as airplane A, airplane B flies 60 degrees west of south at a constant speed of 250 mi/h. Both flights originate at the same time and location. What is the distance between airplanes A and B after they have been flying for 2 hours? Express your answer to the nearest whole number.

97. _____ If the pattern continues, what is the value of x in the third figure?



98. _____ inches The largest circular pizza ever made had a diameter of 122 feet 8 inches. To divide the pizza among 150 contest winners, the chef made straight cuts through the center of the pizza, all equal in length to its diameter. If all the slices had the same area, how many inches were there between slices, measured along the pizza's circumference? Express your answer as a decimal to the nearest tenth.

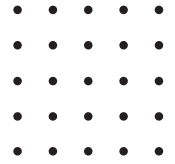
99. _____ If $f(x) = 3x - 4$ and $g(x) = x^2 - 2x + 5$, what is the value of $g(-4) + f(10)$?

100. _____ years The amount of water in a reservoir declines 10% each year. What is the minimum number of whole years it will take until less than half of the original amount of water remains?



Warm-Up 7

101. _____ points In a talent contest involving three finalists, audience members vote for first, second and third place by assigning each a 1, 2 or 3, respectively. When all the votes have been totaled, the finalist with the lowest total wins. If 100 audience members vote and there are no ties in the outcome, what is the greatest number of points that the winner can get?
102. _____ lengths Segments of how many distinct positive lengths can be drawn using pairs of points in this 5×5 evenly spaced grid as endpoints?
103. _____ minutes The time it takes a cube of ice to melt at a certain temperature varies directly with the surface area of the cube. If it takes 3 hours for a 1-foot cube of ice to melt, how many minutes will it take a 1-inch cube of ice to melt if the temperature remains constant? Express your answer as a decimal to the nearest hundredth.
104. _____ The mean score on the last test in Ms. McMean's class of 24 students is 88. If the four lowest scores are excluded, the mean is 94. If the four highest scores are excluded, the mean is 86. What is the absolute difference between the mean of the four highest and four lowest test scores?
105. _____ feet A ball is released from a height of 16 feet, and each time it strikes the ground, it bounces back to a height one-fourth of its previous height. At the instant when the ball strikes the ground for the fifth time, how many total feet has it traveled since it was released? Express your answer as a mixed number.
106. _____ A passenger jet flies a certain distance in 3 hours 30 minutes when traveling west to east. The same distance from east to west requires 4 hours 15 minutes. What is the ratio of the passenger jet's speed going east to the speed going west? Express your answer as a common fraction.
107. _____ lengths In triangle ABC with acute angle BAC, the lengths of sides AB and BC are 15 units and 16 units, respectively. How many possible integer lengths are there for side AC?
108. _____ If $X \# Y = \frac{X}{Y} + XY$, what is the value of $15 \# (6 \# 2)$?
109. _____ % When Jose looked at the clock, he noticed that 20% of the total time from 4:00 p.m. to 5:00 p.m. had elapsed. What percent of the time from 1:00 p.m. to 6:00 p.m. had elapsed, when Jose looked at the clock?
110. _____ combinations In the country of Woodington, three denominations of coins are used to pay for goods and services. Each coin has a value of 1 dollar, 3 dollars or 7 dollars. How many different combinations of coins can be used to pay exactly 15 dollars in Woodington?





Warm-Up 8

111. _____ people At 8:00 a.m., the chairperson of the homecoming committee shared this year's theme with the three other committee members. Within an hour, each of those three committee members told three people who were not on the committee. Every hour after that, each person who had just been told within the previous hour then told three other people who had not yet been told. How many people knew the homecoming theme by the time the first lunch period started at 11:00 a.m.?

112. _____ If $a \blacktriangle b = |a - b|$, then what is the sum of all numbers x such that $(3 \blacktriangle x) \blacktriangle 8 = 2$?

113. _____ integers How many positive integers m are there such that the least common multiple of m and 150 is 300?

114. _____ words Carrick is learning to read. During his first lesson, Carrick read 50 words, and during each lesson thereafter, he read 10 more words than he read during the lesson before. If Carrick had one reading lesson each day for 30 days, what is the total number of words he read in 30 lessons?

115. _____ A standard deck of cards consists of cards numbered 2 through 10 plus a Jack, Queen, King and Ace in each of four different suits. In a particular card game, Jacks, Queens and Kings are each worth 10 points, Aces are worth 11 points and numbered cards are worth face value. Each of four players is dealt three cards, and the winner is the player with the greatest sum of cards of the same suit. If cards have been dealt to the four players as shown, and Austin then is dealt a card from those remaining in the deck, what is the probability that Austin has the winning hand? Express your answer as a common fraction.

AUSTIN	BAILEY	COOPER	DALLAS
8 ♦	Ace ♦	Ace ♥	King ♣
9 ♦	King ♦	3 ♥	9 ♣
?	3 ♠	Queen ♦	5 ♥

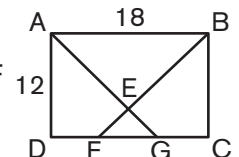
116. _____ If l , m and n are each distinct members of the set $\{2, -\frac{1}{6}, \frac{1}{3}, -3, 12\}$, what is the least possible value of $\frac{l \times m}{n}$?

117. _____ ways In how many ways can each of the digits 3, 5 and 7 be used exactly once to replace X, Y and Z to make the true inequality $0.XY < 0.Z$?

118. _____ For a list of eight positive integers, the mean, median, unique mode and range are 8. What is the greatest integer that could be in this set?

119. _____ Two numbers, x and y , each between 0 and 1, are multiplied. If the tenths digit of x is 1 and the tenths digit of y is 2, what is the greatest possible value of the hundredths digit of the product?

120. _____ units² Quadrilateral ABCD, shown here, is a 12×18 rectangle. Segments BF and AG bisect angles ABC and BAD, respectively, and intersect at E. What is the area of $\triangle EFG$?



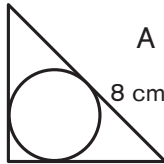


Workout 5

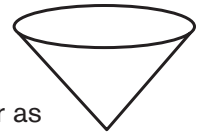
121. _____ inches A photo measures 5 inches wide by 3 inches high. If the total area of the photo is enlarged by 150%, while maintaining the same ratio of width to height, what is the height of the enlarged photo? Express your answer as a decimal to the nearest tenth.

122. _____ The sum of two numbers is 7 and their difference is 18. What is their product? Express your answer as a decimal to the nearest hundredth.

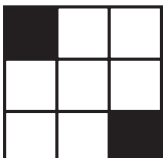
123. _____ cm² A circle is inscribed in an isosceles right triangle with hypotenuse 8 cm. What is the area of the circle? Express your answer as a decimal to the nearest tenth.



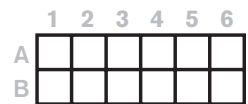
124. _____ A water tank in the shape of a cone, with its point down as shown, has base radius 12 feet and height 8 feet. If the tank is half full by volume. What is the ratio of the depth of the water to the height of the tank? Express your answer as a decimal to the nearest hundredth.



125. _____ inches On her 13th birthday, Moesha's height is exactly 62 inches. If she uses the formula $H(t) = 61 + 2^{0.4t}$ to determine her height H , in inches, t years after her 13th birthday, what should Moesha expect her height to be on her 18th birthday?

126. _____  When filled in correctly with nonzero digits, each row and column in this cross-number grid contains a perfect square. No two rows contain the same perfect square, and no two columns contain the same perfect square. What is the least possible sum of all the digits in the completed grid?

127. _____ ways The figure shows a rectangle divided into 12 congruent squares. In how many ways can each square be colored red, blue or green, so that when the rectangle is rotated 180° every location (for example, row A column 1) contains a square of a different color than the one that was originally in that location?



128. _____ children

Age	Fee
2 & under	No charge
3 to 12	\$3 each
62 & older	\$7 each
All others	\$10 each

On their vacation, the Ship family took a boat ride to Shipwreck Island. The table shows the fees charged by the boat company, based on each passenger's age. The total charge for the 12 family members was \$73. If the number who are "others" exceeds the number who are 62 and older, what is the maximum possible number of children ages 3 to 12?

129. _____ What is the greatest possible sum of the reciprocals of two positive integers with a sum of 11? Express your answer as a common fraction.

130. _____ integers How many positive integers less than or equal to 100 have the same number of odd factors as even factors?



Workout 6

131. _____ If a , b and c are integers between -10 and 10 inclusive such that $a^3 + b^3 = c^3$, what is the greatest possible value of $a + b + c$?

132. _____ The positive integers 1 through 50 are written on 50 cards with one integer on each card. If Matt draws one card at random, what is the probability that the number on the card is a multiple of 6 or 8? Express your answer as a common fraction.

133. _____ units In Figure 1, the vertices of equilateral triangle ABC are connected with segments AB and AC . In Figure 2, the vertices are connected by congruent segments AD , BD and CD that intersect at D . D is the intersection of the medians of triangle ABC . If triangle ABC has side length 1 unit, what is the absolute difference between the sum of the lengths of the two segments in Figure 1 and the sum of the lengths of the three segments in Figure 2? Express your answer in simplest radical form.

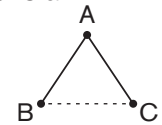


Figure 1

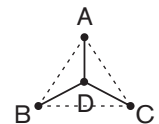


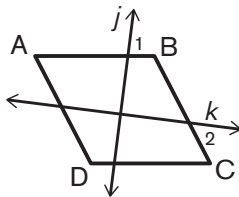
Figure 2

134. _____ % Erik and Alicia went to a restaurant and ordered the same thing for lunch. At the end of the meal, they used different methods to calculate the tip. Erik paid a tip equal to twice the sales tax, and Alicia doubled the final amount on the bill, including the sales tax, and then moved the decimal point in the resulting amount one place to the left. Amazingly, these two methods resulted in the same tip. What was the percent sales tax? Express your answer to the nearest tenth.

135. _____ units² The four vertices of a rectangle also are vertices of a regular hexagon of side length 1 unit. What is the area of the rectangle? Express your answer in simplest radical form.

136. _____ % In a certain state, gas prices increased from \$1.72 to \$3.84 per gallon over a 10-year period. If the prices increased uniformly by the same percent from year to year, what was the annual percent increase? Express your answer to the nearest tenth.

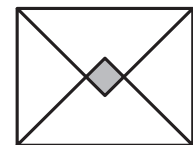
137. _____ degrees The figure shows parallelogram $ABCD$ with $\angle A$ of measure 57 degrees. Line j is perpendicular to line k and also intersects side AB such that $m\angle 1 = 81$ degrees. What is the $m\angle 2$?



138. \$ The table shows the charges associated with shipping 1 to 10 items for one particular retailer. Based on this, what is the least amount a customer can pay per item for shipping?

Items	1	2	3	4	5	6	7	8	9	10
Shipping	\$1.85	\$2.15	\$2.65	\$3.35	\$4.25	\$5.35	\$6.65	\$8.15	\$9.85	\$11.75

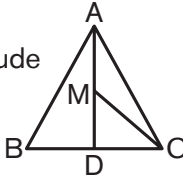
139. _____ % The side lengths of this rectangle are in the ratio 5:7. Two isosceles right triangles are drawn in the rectangle's interior. The hypotenuse of each of these right triangles is one of the longer sides of the rectangle. The shaded region represents what percent of the area of the rectangle? Express your answer to the nearest tenth.



140. _____ If $a \blacktriangledown b = a \cdot b + 3$, what is the absolute difference between $(10 \blacktriangledown 11) \blacktriangledown 12$ and $10 \blacktriangledown (11 \blacktriangledown 12)$?



Warm-Up 9

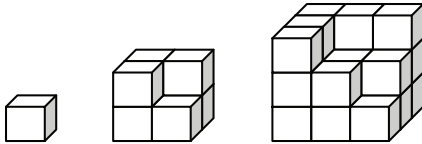
141. _____ Let $f(x) = 3x^2 - 7$ and $g(x) = 2x + 5$. What is the absolute difference between $f(g(-2))$ and $g(f(-2))$?
142. _____ What is the value of $1 - 2 + 3 - 4 + 5 - 6 + \dots + 2013 - 2014 + 2015$?
143. _____ miles Ted and Fred are 60 miles apart and moving toward each other. A carrier pigeon flies back and forth between Ted and Fred without stopping until they meet. If Ted and Fred each maintain a constant speed of 30 mi/h and the pigeon maintains a constant speed of 45 mi/h, what is the total number of miles the pigeon will have flown when Ted and Fred meet?
144. _____ For 12 base x , written 12_x , what is the value of x if $12_x = 3(11_3)$?
145. _____ quadri-
laterals How many different quadrilaterals have vertices with integer coordinates (x, y) such that $0 \leq x \leq 2$ and $0 \leq y \leq 2$?
146. _____ Equilateral triangle ABC has side length 6 units, and M is the midpoint of altitude AD. The length of segment MC expressed as a common fraction in simplest radical form is $\frac{a\sqrt{b}}{c}$ units, where a , b and c are integers. What is the value of $a + b + c$?
- 
147. _____ The perimeter of square A is 28 feet less than the perimeter of square B. The area of square A is 161 square feet less than the area of square B. If x and y are the side lengths of square A and square B, respectively, what is the value of $x + y$?
148. _____ pairings In a certain game, each move consists of pairing tiles of equal value to create a new tile with double the value. For example, a pair of 4-tiles combine to make an 8-tile. Given an unlimited supply of 2-tiles, what is the minimum number of pairings needed to build a tile with a value of 32?
149. _____ The absolute difference between the mean and median of five distinct positive integers is at least 2. If the five integers are 3, 22, 7, 12 and x , with 3 and 22 being the least and greatest values, respectively, what is the sum of all possible values of x ?
150. _____ integers A positive integer k is said to be *divisive* if $k > 10$, all digits of k are nonzero and each digit of k , except the units digit, is a proper divisor (any factor of the number except the number itself) of the digit to its immediate right. Based on this, how many positive integers are divisive?



Warm-Up 10

151. _____ players For each of the 20 basketball games this season, Coach Washington needs to choose 5 players to start. If he doesn't want the same 5 players starting together more than once, what is the minimum number of players the coach needs on his team roster?

152. _____ cubes



If this pattern continues, how many cubes will be in the next figure?

153. _____ In a particular arithmetic sequence, the fourth term is 38, and the sum of the second and seventh terms is 85. What is the value of the fifth term of the sequence?

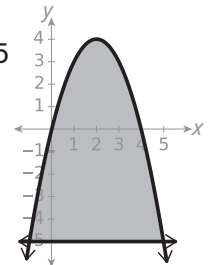
154. _____ units² Let O be the point (0, 0) in the coordinate plane, and let A be the point (3, 7). If B is the point obtained by rotating A 90 degrees counterclockwise about O, then what is the area of triangle ABO?

155. _____ pairs How many ordered pairs of integers (x, y) satisfy the equation $x^2 + y^2 = 2500$?

156. _____ Each letter of the alphabet is assigned a numerical value equal to its position, so that A = 1, B = 2, C = 3, ..., Z = 26. The value of a word is the sum of the values of its letters. For example, the value of THREE is $20 + 8 + 18 + 5 + 5 = 56$. What integer from 1 to 20, inclusive, when spelled out, has the greatest value?

157. _____ ways Spirit Committee members will be chosen from students nominated in each grade. Two students will be chosen from the five nominated 6th graders. Four students will be chosen from the seven nominated 7th graders. Six students will be chosen from the nine nominated 8th graders. In how many different ways can the committee members be chosen?

158. _____ units² The area of the region bounded by the parabola $y = 4x - x^2$ and the line $y = -5$ can be determined using the formula $A = \frac{2}{3}bh$, where b is the length of the horizontal base and h is the vertical distance from the vertex of the parabola to the base. What is the area of this region, shown shaded?



159. _____ fizz In the country of Fizzle, coins come in denominations of 5, 8 and 11 fizz. What is the greatest integer amount that cannot be paid with a whole number of these coins?

160. _____ inches An 8-inch by 12-inch paper napkin is folded in half three times, with each fold resulting in a smaller rectangle. What is the longest possible diagonal for the final rectangle? Express your answer in simplest radical form.



Warm-Up 11

161. _____ in² A right triangle has sides, in inches, measuring $2n$, $n^2 - 1$ and $n^2 + 1$, where n is a positive integer. What is the area of this triangle if its perimeter is 40 inches?

162. _____ integers How many positive three-digit integers have the property that the tens digit is the sum of the units digit and the hundreds digit?

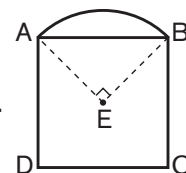
163. _____ An arithmetic sequence of positive integers has a common difference that is three times the first term. If the sum of the first five terms is equal to the absolute difference between the first term and its square, what is the first term in the sequence?

164. _____ in² The area of a rectangle is 168 in^2 , and its perimeter is 62 inches. What is the product of the lengths of the diagonals of this rectangle?

165. _____ If n_k is the k th digit to the right of the decimal point in the decimal representation of $\frac{2240}{1111}$, what is the value of $1000n_{16} + 100n_{13} + 10n_{10} + n_7$?

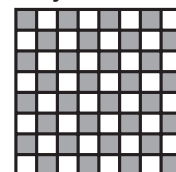
166. _____ What is the greatest prime factor of $6^5 - 4^5 - 2^5$?

167. _____ in² Square ABCD, shown here, has side length 4 inches. The arc from A to B is part of a circle whose center coincides with the center of the square, point E. What is the area of the entire figure? Express your answer in terms of π .



168. _____ quadrants The graph of $f(x) = -x^2 + 8x - 15$ contains points in how many quadrants of the Cartesian coordinate plane?

169. _____ L-pieces What is the maximum number of the L-pieces shown that can be placed entirely on this 8×8 board, with none overlapping, such that the shaded sections of each L-piece are on shaded squares of the board? (Rotating pieces and reflecting pieces are permitted.)



170. _____ steps Justin and Shelby board an escalator as it is moving down. Justin walks down 30 steps and reaches the bottom in 72 seconds, while Shelby walks down 40 steps and reaches the bottom in 60 seconds. If the escalator weren't moving, how many steps would Justin and Shelby each have to walk down to reach the bottom?



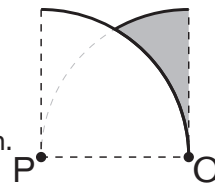
Workout 7

171. _____ If $y = -\frac{5}{4}x + 3$ and $y = -\frac{5}{4}x + 11$ are equidistant from a point with coordinates $(t, 14)$, what is the value of t ? Express your answer as a decimal to the nearest tenth.

172. \$ _____ Priya needs to purchase a camera and a printer. Priya has two coupons, one for 15% off a single item and the other for 20% off any other single item. Priya determines that applying the 20% discount to the printer and the 15% discount to the camera would save her \$0.99 more than if she applied the 15% coupon to the printer and the 20% discount to the camera. What is the absolute difference between the price of the camera and the price of the printer?

173. _____ % The Lemurs are playing the Capybaras in a series of three baseball games. The games alternate between the Lemurs' home field and the Capybaras' home field, with the first game taking place at the Lemurs' field. Each team has a 60% chance of winning a game on its home field. What is the probability, expressed as a percent, that the Lemurs will win at least two of the three games? Express your answer to the nearest tenth.

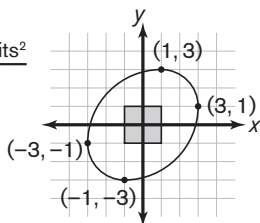
174. _____ units² The figure shows a quarter of unit circle P and a quarter of unit circle Q drawn with the center of each on the circumference of the other. What is the area of the shaded region? Express your answer as a decimal to the nearest hundredth.



175. _____ For a particular sequence, if $a_1 = 2$ and $a_{n+1} = -a_n + 2n$ for $n \geq 1$, what is a_{2016} ?

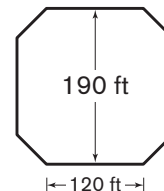
176. _____ A math class is 60% girls. After g girls and 1 boy join the class, it is 75% girls. What is the least possible value of g ?

177. _____ units²



The figure shows a square with vertices $(1, 1)$, $(-1, 1)$, $(-1, -1)$ and $(1, -1)$ and a curve made from four quarter-circular arcs, each centered at a vertex of the square. If the endpoints of the four arcs are $(3, 1)$, $(1, 3)$, $(-3, -1)$ and $(-1, -3)$, what is the area inside the curve? Express your answer in terms of π .

178. _____ feet This octagon is classified as *isogonal* since each of its vertices is between one short side and one long side, and its interior angles are all congruent. If the long sides have length 120 feet and the distance between two parallel long sides is 190 feet, what is the perimeter of the octagon? Express your answer to the nearest whole number.



179. _____ terms All terms of an arithmetic sequence are integers. If the first term is 13, the last term is 77 and the sequence has n terms, what is the median of all possible values of n ?

180. _____ Will and Ian each randomly choose a positive divisor of 20. What is the probability that the least common multiple of their chosen numbers is 20? Express your answer as a common fraction.



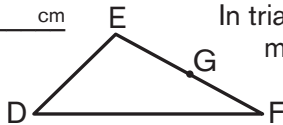
Workout 8

181. _____ divisors How many positive integer divisors does 2016 have?

182. _____ If $f(x) = x^x$ and $f(f(f(2))) = 2^k$, what is the integer value of k ?

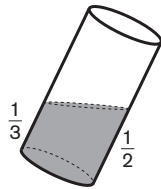
183. _____ units² Circle O, with radius 8 units, and circle W, with radius 3 units, are externally tangent. The circles are also tangent to line m at distinct points B and L, respectively. What is the area of quadrilateral BOWL? Express your answer in simplest radical form.

184. _____ cm In triangle DEF, DE = 6 cm, DF = 9 cm and DG = 7 cm, where G is the midpoint of side EF. What is the length of side EF? Express your answer in simplest radical form.



185. _____ The digits 1 through 7 are each used once to write three prime numbers. Two of these primes have two digits each and one has three digits. What is the greatest possible value of the three-digit prime number?

186. _____ in³ Dan tilts his soda can until the soda is halfway up the can at its highest point and a third of the way up the can at its lowest point. If the can is 4.5 inches tall with a 2.5-inch diameter, how many cubic inches of soda are in the can? Express your answer as a decimal to the nearest tenth.



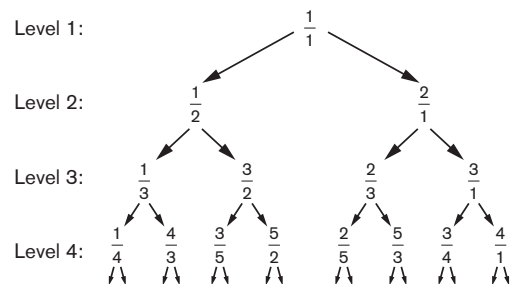
187. _____ triangles Four rows from a sheet of isometric dot paper, where the rows contain three, two, three and two points, respectively, are shown. How many distinct non-equilateral triangles can be drawn with each vertex at a point in this array?



188. _____ Two positive integers not exceeding 100 have the property that their sum is divisible by their difference. What is the greatest possible value of their difference?

189. _____ If Pascal's Triangle is folded along its vertical line of symmetry and then unfolded, what is the sum of the seven numbers on the fold line that are of least value?

190. _____ The figure shows part of an infinite tree diagram in which each fraction $\frac{a}{b}$ has the fractions $\frac{a}{a+b}$ and $\frac{a+b}{b}$ written below it, starting at Level 1 with $\frac{1}{1}$. For what integer n does Level n contain the fraction $\frac{31}{4159}$?





Warm-Up 12

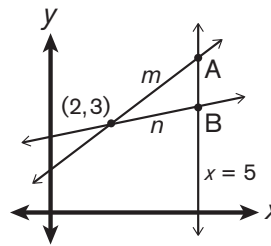
191. _____ When three fair dice are rolled, what is the probability that the product of the three numbers rolled is a prime number? Express your answer as a common fraction.

192. _____ inches A tree is 8 feet tall when planted. After one year, its height has increased by one-sixth of its original height. At the end of the second year, its height is 37.5% greater than its original height. How many inches did the tree grow during the second year?

193. _____ Kenyatta has some quarters on a table, each showing either heads or tails. Kenyatta chooses 20% of the quarters that show heads and 10% of the quarters that show tails and then turns over each of the chosen quarter one time. After she does this, exactly half of the quarters on the table show heads. Before Kenyatta turned the quarters over, what was the ratio of the number of quarters that showed heads to the number that showed tails? Express your answer as a common fraction.

194. _____ minutes If three machines can fill 80 boxes in 2 hours, how many minutes will it take five machines to fill 150 boxes?

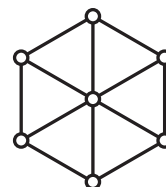
195. _____ units Lines m and n both pass through the point $(2, 3)$ as shown. Line m has slope $\frac{3}{4}$, and line n has slope $\frac{1}{5}$. If lines m and n intersect the line $x = 5$ at the points A and B , respectively, what is the distance AB ? Express your answer as a common fraction.



196. _____ What integer is closest in value to $\frac{1}{\sqrt{102} - \sqrt{98}}$?

197. \$ _____ Victor pays \$1.00 for a bottle of soda at his local market. Each time Victor returns three empty bottles, he earns a free bottle of soda. What is the minimum amount Victor must spend to get 10 bottles of soda?

198. _____ ways The figure shows a regular hexagon and seven circles, one at each vertex and another at the hexagon's center. Three diagonals are drawn through the center of the hexagon, connecting opposite vertices. In how many ways can each of the circles be colored either red or blue so that no three collinear circles are the same color?



199. _____ Four husband-and-wife couples attend a show. From these four couples, two people are randomly selected when the performer asks for volunteers from the audience. What is the probability that the two who are selected are a married couple? Express your answer as a common fraction.

200. _____ In a quadratic equation $Ax^2 + Bx + C = 0$, A , B and C are integers whose only common factor is 1. The roots of the equation are $\frac{2}{3}$ and 4. If $A > 0$, what is the value of $A + B + C$?



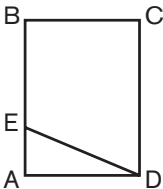
Warm-Up 13

201. _____ minutes For the first 4 miles of a 6-mile race, it took Jenny an average of $10\frac{1}{3}$ minutes to run each mile. If she wants to run the entire race in exactly 1 hour, how many minutes, on average, must she take to run each mile for the remainder of the race? Express your answer as a mixed number.

202. _____ hours Two 12-hour clocks were started simultaneously, both showing the same time. Clock A loses 15 minutes each hour, and clock B gains 15 minutes each hour. How many hours pass before the first instance when that Clock A again shows the same time as Clock B?

203. _____ What is the value of $f(f(f(19) + 1) + 1)$ if $f(x) = \begin{cases} x^2 & \text{if } x \text{ is even} \\ \frac{x-3}{2} & \text{if } x \text{ is odd} \end{cases}$?

204. _____ Rectangle ABCD, shown here, has point E on side AB such that the ratio of the area of trapezoid EBCD to triangle AED is 5:1. What is the value of the ratio AE:EB? Express your answer as a common fraction.



205. _____ in³ Kendra will make a box with an open top by cutting, folding and taping a 12-inch by 16-inch rectangular piece of cardboard. She will begin by cutting from each corner of the flat cardboard a square with a side length that is a multiple of 0.5 inch. What is the maximum possible volume of the box? Express your answer as a decimal to the nearest tenth.

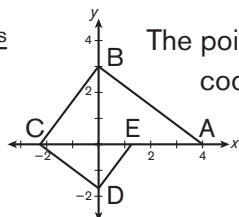
206. _____ What is the sum of the coefficients of the terms when $(2x + y)^5$ is expanded?

207. _____ In a certain game of darts, a dart that lands on the bull's-eye scores 11 points, and a dart that lands any other place on the board scores 7 points. A player throws three darts, and the probability that the player hits the bull's-eye is 50% on each throw, independent of previous results. Given that each dart lands on the board, and at least one dart lands on the bull's-eye, what is the probability that the total number of points scored will be prime? Express your answer as a common fraction.

208. _____ integers How many 4-digit integers have their digits in strictly ascending order?

209. (_____ , _____) $P(-3, -2)$ is reflected over the line $y = -x$ and then translated 4 units right and 1 unit down. What are the coordinates of the final image of P? Express your answer as an ordered pair.

210. _____ units The points $A(4, 0)$, $B(0, 3)$, $C(c, 0)$, $D(0, d)$ and $E(e, 0)$ are graphed on the coordinate plane as shown. If $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{CD}$ and $\overline{CD} \perp \overline{DE}$, what is the combined length of the four segments? Express your answer as a common fraction.





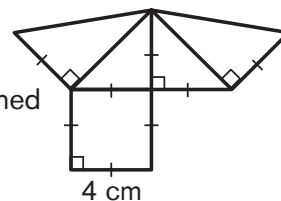
Warm-Up 14

211. _____ If $3^x = y^2$, and $9^{5x} = y^z$, what is the value of z ?

212. _____ What is the value of the sum $\frac{3+6}{9} + \frac{12+15}{18} + \frac{21+24}{27} + \dots + \frac{48+51}{54}$? Express your answer as a common fraction.

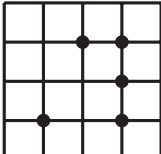
213. _____ members Namakshi wants to arrange her marching band in a rectangular array with the same number of band members in each row. But when she tried putting them in 3 rows, she had 1 band member left over; in 5 rows, she had 2 members left over; in 7 rows, she had 3 members left over; and in 9 rows, she had 4 members left over. What is the least number of members that could be in the band?

214. _____ cm^3 Three pyramids, each created using the net shown here, can be combined to form a cube. What is the volume of the cube?



215. _____ In a certain sequence of numbers, every term after the second term is equal to the sum of the two preceding terms. But the 99th term of the sequence is 7 and the 102nd term is 69, what is the 100th term of the sequence?

216. _____ cents Stacey has some pennies, nickels and dimes divided between her left and right pockets. She has the same number of coins in each pocket, and the coins in her two pockets add to the same total value. Stacey does not have the same number of pennies in each pocket. What is the minimum total value that she could have in both pockets combined?

217. _____ units  In this grid composed of 16 unit squares, the five dots can be labeled with the letters A through E so that $AB < BC < CD < DE$. What is the length of segment CE?

218. _____ What is the probability that a randomly chosen two-digit number containing at least one 1 or 9 is prime? Express your answer as a common fraction.

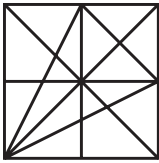
219. _____ minutes Alfonso, Benjamin and Carmen walk at speeds of 2, 6 and 8 feet per second, respectively. The three friends start walking together at the same time in the same direction around an oval track that measures 440 yards around. After how many minutes are the three friends next at the same exact location on the track at the same exact time?

220. _____ For each positive integer $n \leq 55$, if $S(n)$ is the sum of the positive integer divisors of n , what is the greatest possible value of $S(n)$?



Counting Stretch

221. _____ Hazel wrote the integers 1 through 321 on the board. How many total digits did she write?

222. _____ triangles  How many triangles of any size are in this figure?

223. _____ ways In how many ways can one knife, one fork and one spoon be distributed, in any order, to three people, if each person is given 0, 1, 2 or 3 utensils?

224. _____ ways Using pennies, nickels, dimes and quarters, how many ways can you make 67 cents?

225. _____ scores In the game Fortrix, a player can earn 3, 7 or 11 points on a turn. How many different scores are possible for a single player after six turns?

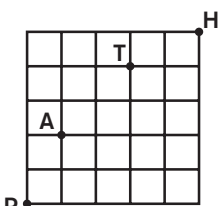
226. _____ integers How many 3-digit integers are divisible by both 5 and 17?

227. _____ integers How many positive integers less than 40 are relatively prime to both 7 and 10?

228. _____ palin-
dromes How many palindromes are between 9 and 1009?

229. _____ paths In the 3×3 grid shown, a path can begin in any cell and can pass through a cell more than once. How many such paths spell ROTOR?

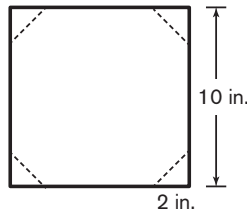
R	O	R
O	T	O
R	O	R

230. _____ paths  Moving only up and right, how many paths from P to H pass through A and T?

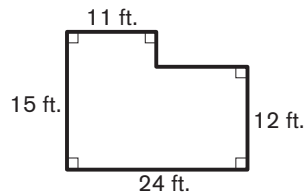


Area Stretch

231. _____ % Norm has a square sheet of paper with 10-inch sides. Along each side, he makes a mark 2 inches from each corner. He then draws a line segment connecting the two marks near each corner. Finally, he cuts along each line segment, removing a triangle from each corner of the square and creating an octagon. What percentage of the area of the square is the area of the octagon?



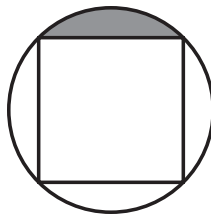
232. _____ ft^2 The figure shows an office floor plan. How many square feet does this office occupy?



233. _____ m^2 A running track consists of two parallel straight segments, each 100 meters long, connected by two semicircular stretches, each with inner diameter 50 meters. What is the total area enclosed by the running track? Express your answer to the nearest hundred.

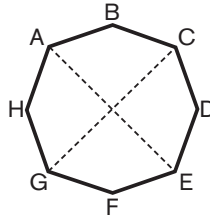
234. _____ units^2 What is the greatest possible area of a concave pentagon in the coordinate plane with vertices $(-2, 0)$, $(2, 0)$, $(2, 10)$, $(0, 6)$ and $(-2, 10)$?

235. _____ units^2 A square is inscribed in a circle of radius 4 units. The square divides the interior of the circle into five regions, four of which lie outside the square. What is the area of the shaded region? Express your answer in terms of π .

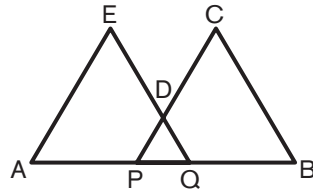


236. _____ in² Amy marks two points A and B that are 4 inches apart. She draws one circle that has segment AB as a diameter. She then draws a larger circle, which overlaps the first circle, such that the arc from A to B along its circumference is a quarter-circle. What is the total area covered by the two circles? Express your answer in terms of π .

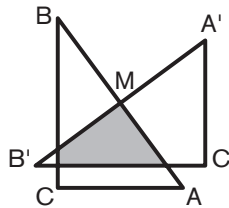
237. _____ units² In convex octagon ABCDEFGH, shown here, each side has length 6 units, and diagonals AE and CG have length 16 units. If the octagon is symmetric across both diagonals AE and CG, what is its area? Express your answer in simplest radical form.



238. _____ units² In this figure, $AE = EQ = BC = CP = 10$ units, and $AQ = BP = 12$ units. The points A, P, Q and B are collinear. If the perimeter of the concave pentagon ABCDE is 52 units, what is its area? Express your answer as a common fraction.



239. _____ units² Right triangle ABC with $AC = 3$ units, $BC = 4$ units and $AB = 5$ units is rotated 90° counterclockwise about M, the midpoint of side AB, to create a new right triangle A'B'C'. What is the area of the shaded region where triangles ABC and A'B'C' overlap? Express your answer as a common fraction.



240. _____ units In right triangle ABC, $\angle C$ is a right angle, $AC = 10$ units and $BC = 24$ units. If a point X is located inside triangle ABC so that the distance from X to side AB is twice the distance from X to side AC, and the distance from X to side AC is twice the distance from X to side BC, what is the distance from X to side AB? Express your answer as a common fraction.



Modular Arithmetic Stretch

Modular arithmetic is a system of integer arithmetic that enables us obtain information and draw conclusions about large quantities and calculations. It would be extremely helpful, for instance, when asked to find the units digit of 2^{2015} if we didn't really have to calculate the value of the expression to get that information. Modular arithmetic allows us to do just that!

THE BASICS:

The simplest example of modular arithmetic is commonly referred to as “clock arithmetic.” Suppose it is 3 o'clock now and I want to know what time it will be in 145 hours. We could count from 3 o'clock for 145 consecutive hours. We certainly wouldn't be expected to count 145 hours starting with 3 o'clock. Suppose we did counting the hours from 3 o'clock. What happens when we get to 12 o'clock? We continue counting but begin a new 12-hour cycle. Instead of counting 145 hours, we can just see how many of these 12-hour cycles we'd go through counting 145 hours. More importantly, we need to determine how many hours would remain after making it through the last full 12-hour cycle.

In this example, the value 12 is called the **modulus** and what is left over is called the remainder. In this case, we can determine fairly quickly that there are 12 full 12-hour cycles in 145 hours, with a remainder of 1 hour (since $12 \times 12 = 144$ and $145 - 144 = 1$).

$$\begin{array}{l} \text{Standard arithmetic: } 145 = 12 \times 12 + 1 \\ \text{Modular arithmetic we write: } 145 \equiv 1 \pmod{12} \end{array}$$



Read “145 is congruent to 1 modulo 12”

The remainder of 1 tells me that it will be the same time 145 hours after 3 o'clock that it will be 1 hour after 3 o'clock. And that time is 4 o'clock.

Here's another example of modular arithmetic. Suppose today is Tuesday. What day of the week will it be 417 days from now? Since the days of the week are on a 7-day repeating cycle, the modulus here is 7. If we divide 417 by 7, we get

$$\begin{array}{l} \text{Standard arithmetic: } 417 = 59 \times 7 + 4 \\ \text{Modular arithmetic we write: } 417 \equiv 4 \pmod{7} \end{array}$$

Thus, 417 days from Tuesday will be the same day of the week as 4 days from Tuesday, Saturday.

TRY THESE

241. _____ If the current month is July, what month will it be in 152 months?

242. _____ a.m. If the time is currently 8 a.m., what time will it be in 255 hours?
p.m. Circle a.m. or p.m. in answer blank.

243. _____ m Jennie goes out every morning and jogs on the school track. The track is 400 meters around. If Jennie runs 5310 meters then how far will she be from where she started once she finished her run?

MODULAR ADDITION: What is the remainder when $9813 + 7762 + 11252$ is divided by 10?

$$\begin{aligned} 9813 + 7762 + 11252 &= (981 \times 10 + 3) + (776 \times 10 + 2) + (1125 \times 10 + 2) \\ &= (981 + 776 + 1125) \times 10 + (3 + 2 + 2) \end{aligned}$$

Since we are only interested in the remainder, we need only focus on the last part. We see that the remainder is $3 + 2 + 2 = 7$. Written in modular arithmetic notation it would look like this:

$$9813 + 7762 + 11252 \equiv 3 + 2 + 2 \equiv 7 \pmod{10}$$

MODULAR MULTIPLICATION: What is the remainder when 9813×7762 is divided by 10?

$$\begin{aligned} 9813 \times 7762 &= (981 \times 10 + 3) \times (776 \times 10 + 2) \\ &= (981 \times 776 \times 10^2) + (981 \times 2 \times 10) + (776 \times 3 \times 10) + (3 \times 2) \end{aligned}$$

The first three terms are multiples of 10, and once again last term is the remainder $3 \times 2 = 6$. Written in modular arithmetic notation would look like this:

$$9813 \times 7762 \equiv 3 \times 2 \equiv 6 \pmod{10}$$

MORE MOD SHORTCUTS: There are many useful applications of modular arithmetic. Here are just a few more.

- Consider the powers of 3: $3^0 = 1$; $3^1 = 3$; $3^2 = 9$; $3^3 = 27$; $3^4 = 81$; $3^5 = 243$; $3^6 = 729$
Notice that the units digits are repeated every four powers of 3, so the modulus is 4. Repeating units digits correspond to remainders 1, 2, 3 and 0.
- Suppose you want the unit digit of 3^{53} . First, we note that $53 \equiv 1 \pmod{4}$ since the remainder 1 corresponds to units digit 3, thus, the expansion of 3^{53} has a units digit of 3.
- The smallest number that has remainder 1 when divided by 2 and 3 is 7. Why?
 $1 \equiv 7 \pmod{2}$ and $1 \equiv 7 \pmod{3}$

MODULAR ARITHMETIC PRACTICE

244. _____ What is the last digit of 2^{2015} ?
245. _____ What is the value of 122×71 modulo 11?
246. _____ What is the remainder when $5981 \times 8162 \times 476$ is divided by 5.
247. _____ Jon has 29 boxes of donuts with 51 donuts in each box. He wants to divide them into groups of a dozen each. Once he groups them again, how many donuts will be left over?
248. _____ What is the least number that leaves a remainder of 6 when divided by 7 and 11?
249. _____ When organizing her pencils, Faith notices that when she puts them in groups of 3, 4, 5, or 6, she always has exactly one pencil left over. If Faith has between 10 and 100 pencils, how many pencils does she have?
250. _____ When organizing her pens, Faith notices that when she puts them in groups of 3, 4, 5, or 6, she is always one pen short of being able to make full groups. If Faith has between 10 and 100 pens, how many pens does she have?

FORMS OF ANSWERS

The following rules explain acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Competition answers will be scored in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lb 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

All answers must be expressed in simplest form. A “common fraction” is to be considered a fraction in the form $\pm \frac{a}{b}$, where a and b are natural numbers and $\text{GCF}(a, b) = 1$. In some cases the term “common fraction” is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not have a common factor. A simplified “mixed number” (“mixed numeral,” “mixed fraction”) is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N , a and b are natural numbers, $a < b$ and $\text{GCF}(a, b) = 1$. Examples:

Problem: What is $8 \div 12$ expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$
Problem: What is $12 \div 8$ expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}, 1\frac{1}{2}$
Problem: What is the sum of the lengths of the radius and the circumference of a circle of diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π ? *Answer:* $\frac{1+2\pi}{8}$
Problem: What is $20 \div 12$ expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}, \frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Acceptable Simplified Forms: $\frac{7}{2}, \frac{3}{\pi}, \frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}, \frac{1}{3}, 3.5, 2:1$

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: What is $\sqrt{15} \times \sqrt{5}$ expressed in simplest radical form? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., “How many dollars...,” “How much will it cost...,” “What is the amount of interest...”) should be expressed in the form (\$) $a.bc$, where a is an integer and b and c are digits. The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c are both zero, in which case they both may be omitted. Answers in the form (\$) $a.bc$ should be rounded to the nearest cent, unless otherwise specified. Examples:

Acceptable Forms: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π , $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the “rounding” a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:

Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3

Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 or 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form. Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

VOCABULARY AND FORMULAS

The following list is representative of terminology used in the problems but should not be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute difference	decimal	infinite series
absolute value	degree measure	inscribe
acute angle	denominator	integer
additive inverse (<i>opposite</i>)	diagonal of a polygon	interior angle of a polygon
adjacent angles	diagonal of a polyhedron	interquartile range
algorithm	diameter	intersection
alternate exterior angles	difference	inverse variation
alternate interior angles	digit	irrational number
altitude (<i>height</i>)	digit-sum	isosceles
apex	direct variation	kite
area	dividend	lateral edge
arithmetic mean	divisible	lateral surface area
arithmetic sequence	divisor	lattice point(s)
base 10	dodecagon	LCM
binary	dodecahedron	linear equation
bisect	domain of a function	mean
box-and-whisker plot	edge	median of a set of data
center	endpoint	median of a triangle
chord	equation	midpoint
circle	equiangular	mixed number
circumference	equidistant	mode(s) of a set of data
circumscribe	equilateral	multiple
coefficient	evaluate	multiplicative inverse (<i>reciprocal</i>)
collinear	expected value	natural number
combination	exponent	nonagon
common denominator	expression	numerator
common divisor	exterior angle of a polygon	obtuse angle
common factor	factor	octagon
common fraction	factorial	octahedron
common multiple	finite	ordered pair
complementary angles	formula	origin
composite number	frequency distribution	palindrome
compound interest	frustum	parallel
concentric	function	parallelogram
cone	GCF	Pascal's Triangle
congruent	geometric mean	pentagon
convex	geometric sequence	percent increase/decrease
coordinate plane/system	height (<i>altitude</i>)	perimeter
coordinates of a point	hemisphere	permutation
coplanar	heptagon	perpendicular
corresponding angles	hexagon	planar
counting numbers	hypotenuse	polygon
counting principle	image(s) of a point(s)	polyhedron
cube	(<i>under a transformation</i>)	polynomial
cylinder	improper fraction	prime factorization
decagon	inequality	prime number

principal square root	revolution	supplementary angles
prism	rhombus	system of equations/inequalities
probability	right angle	tangent figures
product	right circular cone	tangent line
proper divisor	right circular cylinder	term
proper factor	right polyhedron	terminating decimal
proper fraction	right triangle	tetrahedron
proportion	rotation	total surface area
pyramid	scalene triangle	transformation
Pythagorean Triple	scientific notation	translation
quadrant	sector	trapezoid
quadrilateral	segment of a circle	triangle
quotient	segment of a line	triangular numbers
radius	semicircle	trisect
random	semiperimeter	twin primes
range of a data set	sequence	union
range of a function	set	unit fraction
rate	significant digits	variable
ratio	similar figures	vertex
rational number	simple interest	vertical angles
ray	slope	volume
real number	slope-intercept form	whole number
reciprocal (<i>multiplicative inverse</i>)	solution set	x-axis
rectangle	space diagonal	x-coordinate
reflection	sphere	x-intercept
regular polygon	square	y-axis
relatively prime	square root	y-coordinate
remainder	stem-and-leaf plot	y-intercept
repeating decimal	sum	

The list of formulas below is representative of those needed to solve MATHCOUNTS problems but should not be viewed as the only formulas that may be used. Many other formulas that are useful in problem solving should be discovered and derived by Mathletes.

CIRCUMFERENCE

Circle $C = 2 \times \pi \times r = \pi \times d$

AREA

Circle $A = \pi \times r^2$

Square $A = s^2$

Rectangle $A = l \times w = b \times h$

Parallelogram $A = b \times h$

Trapezoid $A = \frac{1}{2}(b_1 + b_2) \times h$

Rhombus $A = \frac{1}{2} \times d_1 \times d_2$

Triangle $A = \frac{1}{2} \times b \times h$

Triangle $A = \sqrt{s(s-a)(s-b)(s-c)}$

Equilateral triangle $A = \frac{s^2\sqrt{3}}{4}$

SURFACE AREA AND VOLUME

Sphere $SA = 4 \times \pi \times r^2$

Sphere $V = \frac{4}{3} \times \pi \times r^3$

Rectangular prism $V = l \times w \times h$

Circular cylinder $V = \pi \times r^2 \times h$

Circular cone $V = \frac{1}{3} \times \pi \times r^2 \times h$

Pyramid $V = \frac{1}{3} \times B \times h$

Pythagorean Theorem $c^2 = a^2 + b^2$

Counting/
Combinations ${}_n C_r = \frac{n!}{r!(n-r)!}$

ANSWERS

In addition to the answer, we have provided a difficulty rating for each problem. Our scale is 1-7, with 7 being the most difficult. These are only approximations, and how difficult a problem is for a particular student will vary. Below is a general guide to the ratings:

Difficulty 1/2/3 - One concept; one- to two-step solution; appropriate for students just starting the middle school curriculum.

4/5 - One or two concepts; multistep solution; knowledge of some middle school topics is necessary.

6/7 - Multiple and/or advanced concepts; multistep solution; knowledge of advanced middle school topics and/or problem-solving strategies is necessary.

Warm-Up 1

Answer	Difficulty
1. 200	(1)
2. 59	(2)
3. 1080	(3)
4. 31.68	(2)
5. 8	(3)
6. 88	(2)
7. 7	(2)
8. 277	(2)
9. 13	(3)
10. -800	(4)

Warm-Up 2

Answer	Difficulty
11. 1	(1)
12. 2	(2)
13. 12	(2)
14. 5/3	(3)
15. 36,300	(3)
16. 20	(5)
17. 6	(2)
18. 113	(3)
19. 225	(4)
20. 16/47	(4)

Warm-Up 3

Answer	Difficulty
21. 4	(2)
22. 223,200	(2)
23. 1/6	(4)
24. 20	(4)
25. 1/3	(5)
26. 81	(4)
27. 7.5	(4)
28. 9	(3)
29. 107	(5)
30. 10	(4)

Workout 1

Answer	Difficulty
31. 3.6	(2)
32. 28	(2)
33. 64.5	(3)
34. 1.45	(5)
35. 13	(2)
36. 7.2	(3)
37. 32.6	(4)
38. 5	(4)
39. 1×10^9	(4)
40. 12	(4)

Workout 2

Answer	Difficulty
41. 6	(3)
42. 19.2	(4)
43. 5 or 5.00	(3)
44. 42.9	(4)
45. 2.70	(4)
46. 16.4	(3)
47. 0.8	(2)
48. 40	(3)
49. 72	(4)
50. 1*	(3)

Warm-Up 4

Answer	Difficulty
51. 50	(2)
52. $18\frac{1}{3}$	(2)
53. red	(2)
54. 8	(5)
55. 259	(4)
56. 5940	(3)
57. 55	(4)
58. 48	(4)
59. 550	(5)
60. 20	(4)

* The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.

Warm-Up 5

Answer	Difficulty			
61. 34	(4)	66. 15	(3)	
62. 24	(3)	67. $1/5$	(4)	
63. 25	(3)	68. Monday	(3)	
64. 75 or 75.00	(3)	69. 6	(4)	
65. 432	(4)	70. 15	(4)	

Warm-Up 6

Answer	Difficulty			
71. 2	(4)	76. -27	(4)	
72. 15	(3)	77. $328\frac{1}{8}$	(4)	
73. 60	(3)	78. 3	(4)	
74. 64,000	(5)	79. $511/512$	(4)	
75. 88	(3)	80. 60 or 60.00	(3)	

Workout 3

Answer	Difficulty			
81. 25	(3)	86. 12	(5)	
82. 4.05	(3)	87. 96	(5)	
83. 240	(4)	88. 6.83	(4)	
84. 5.6	(4)	89. 1.75	(5)	
85. 0.025	(3)	90. 24	(3)	

Workout 4

Answer	Difficulty			
91. 15	(3)	96. 1091	(5)	
92. 0.9	(4)	97. 3	(3)	
93. 7.5	(4)	98. 30.8	(3)	
94. 3.04	(4)	99. 55	(2)	
95. 157.5	(4)	100. 7	(4)	

Warm-Up 7

Answer	Difficulty			
101. 199	(3)	106. $17/14$	(3)	
102. 14	(3)	107. 25	(6)	
103. 1.25	(3)	108. 226	(3)	
104. 40	(4)	109. 64	(3)	
105. $26\frac{5}{8}$	(3)	110. 10	(3)	

Warm-Up 8

Answer	Difficulty			
111. 121	(3)	116. -144	(4)	
112. 12	(4)	117. 3	(3)	
113. 6	(4)	118. 14	(5)	
114. 5850	(4)	119. 5	(4)	
115. $5/41$	(4)	120. 9	(5)	

Workout 5

Answer	Difficulty		
121. 4.7	(3)	126. 26	(4)
122. -68.75	(4)	127. 46,656	(5)
123. 8.6	(4)	128. 4	(5)
124. 0.79	(5)	129. 11/10	(3)
125. 65	(3)	130. 25	(4)

Workout 6

Answer	Difficulty		
131. 20	(3)	136. 8.4	(5)
132. 6/25	(4)	137. 48	(5)
133. $2 - \sqrt{3}$ or $-\sqrt{3} + 2$	(5)	138. 0.84	(2)
134. 11.1	(5)	139. 5.7	(5)
135. $\sqrt{3}$	(5)	140. 6	(3)

Warm-Up 9

Answer	Difficulty		
141. 19	(4)	146. 12	(5)
142. 1008	(4)	147. 23	(4)
143. 45	(3)	148. 15	(4)
144. 10	(4)	149. 23	(4)
145. 78	(5)	150. 22	(4)

Warm-Up 10

Answer	Difficulty		
151. 7	(4)	156. 17	(3)
152. 50	(3)	157. 29,400	(4)
153. 47	(4)	158. 36	(5)
154. 29	(4)	159. 17	(3)
155. 20	(5)	160. $\sqrt{145}$	(4)

Warm-Up 11

Answer	Difficulty		
161. 60	(4)	166. 7	(5)
162. 45	(3)	167. $2\pi + 12$ or $12 + 2\pi$	(5)
163. 36	(5)	168. 3	(4)
164. 625	(5)	169. 16	(4)
165. 2016	(5)	170. 90	(4)

Workout 7

Answer	Difficulty		
171. -5.6	(5)	176. 6	(5)
172. 19.80	(4)	177. $10\pi - 4$ or $-4 + 10\pi$	(5)
173. 55.2	(5)	178. 678	(4)
174. 0.17	(5)	179. 9	(5)
175. 2014	(5)	180. 5/12	(5)

Workout 8

Answer	Difficulty		
181. 36	(3)	186. 9.2	(5)
182. 2048	(4)	187. 93	(4)
183. $22\sqrt{6}$	(6)	188. 66	(4)
184. $\sqrt{38}$	(5)	189. 1275	(4)
185. 547	(4)	190. 145	(6)

Warm-Up 12

Answer	Difficulty		
191. $1/24$	(4)	196. 5	(5)
192. 20	(3)	197. 7 or 7.00	(3)
193. $4/3$	(5)	198. 54	(5)
194. 135	(4)	199. $1/7$	(4)
195. $33/20$	(4)	200. -3	(4)

Warm-Up 13

Answer	Difficulty		
201. $9\frac{1}{3}$	(3)	206. 243	(5)
202. 24	(3)	207. $3/7$	(3)
203. 16	(4)	208. 126	(5)
204. $1/2$	(4)	209. (6, 2)	(5)
205. 192.5	(5)	210. $875/64$	(6)

Warm-Up 14

Answer	Difficulty		
211. 20	(4)	216. 90	(5)
212. $191/20$	(4)	217. 1	(4)
213. 157	(4)	218. $13/34$	(4)
214. 64	(3)	219. 11	(4)
215. 31	(4)	220. 124	(5)

Counting Stretch

Answer	Difficulty		
221. 855	(2)	226. 10	(3)
222. 56	(3)	227. 14	(3)
223. 27	(3)	228. 100	(3)
224. 87	(3)	229. 64	(4)
225. 13	(4)	230. 54	(4)

Area Stretch

Answer	Difficulty		
231. 92	(3)	236. $4 + 8\pi$	(5)
		or $8\pi + 4$	
232. 321	(2)		
233. 7000	(3)	237. $128 + 32\sqrt{2}$	(6)
		or $32\sqrt{2} + 128$	
234. 32	(3)	238. $357/4$	(6)
235. $4\pi - 8$	(4)	239. $9/4$	(7)
		or $-8 + 4\pi$	
		240. $240/37$	(6)

Modular Arithmetic Stretch

Answer	Difficulty		
241. March	(3)	246. 2	(4)
242. 11:00 p.m.	(3)	247. 3	(4)
243. 110	(4)	248. 83	(5)
244. 8	(5)	249. 61	(6)
245. 5	(4)	250. 59	(6)